FRM 考试公式表

$$NPV = \sum_{t=0}^{N} \frac{CF_t}{(1 + discount\,rate)^t}$$

Foundations of Risk Management

-

- $\textbf{WACC} = W_e \times \text{Cost of equity} + W_d \times \text{Cost of debt}$

  Cost of equity = risk-free rate +beta ×equity risk premium (CAPM)

  Cost of debt = cost of borrowing × (1-marginal tax rate)

  $$\textbf{beta}_i = \frac{Cov(R_i, R_M)}{\sigma_M^2}$$

- **Measures of systematic risk:**

- **Arbitrage Pricing Theory (APT):** $E(R_i) = R_F + \beta_{i1}RP_1 + \beta_{i2}RP_2 + \ldots + \beta_{iK}RP_K$

- **Measures of Performance:**

  $$\text{Jensen's alpha} = E(R_P) - \{R_F + \beta_P[E(R_M) - R_F]\}$$

  $$\text{Treynor measure} = \left[\frac{E(R_P) - R_F}{\beta_P}\right]$$

  $$\text{Sharpe measure} = \left[\frac{E(R_P) - R_F}{\sigma_P}\right]$$

  $$\text{Sortino Ratio} = \frac{E(R_p) - R_{min}}{\text{semi-standard deviation}}$$

  $$\text{Information Ratio (IR)} = \left[\frac{E(R_P) - E(R_B)}{\text{tracking error}}\right]$$

- **Financial Disaster:**

Barings: rogue trader, Nick Leeson, took speculative derivative positions (Nikkei 225 futures) in an attempt to cover trading losses; Leeson had dual responsibilities of trading and supervising settlement operations, allowing him to hide trading losses; lessons include separation of duties and management oversight

Meltallgesellschaft: short-term futures contracts used to hedge long-term exposure in the petroleum markets; stack-and-roll hedging strategy; marking to market on futures caused

huge cash flow problems.

Long-Term Capital Management: hedge fund that used relative value strategies with enormous

amounts of leverage; when Russia defaulted on its debt in 1998, the increase in yield spreads caused huge losses and enormous cash flow problems from realizing marking to market

losses; lessons include lack of diversification, model risk, leverage, and funding and trading liquidity risks.

Quantitative Analysis

$$P(I|O) = \frac{P(O|I)}{P(O)} \times P(I)$$

- **Baye's Formula:**
- **Expected value:** $E(X) = \sum P(x_i)x_i$

  Variance: $\text{Variance}(X) = E\left[(X-\mu)^2\right]$

$$\text{Corr}(R_i, R_j) = \frac{\text{Cov}(R_i, R_j)}{\sigma(R_i)\sigma(R_j)}$$

$$\text{Cov}(R_i, R_j) = E\{[R_i - E(R_i)] \times R_j - E(Rj)\}$$

- **Covariance:**

  Correlation:

- **Skewness** refers to the **symmetrization** of distribution. Skewness of normal distribution is 0. A **positively skewed** distribution is right tail. A **negatively skewed** distribution is left tail.
- **Kurtosis** measures the distribution is more or less "peaked" than normal

$$z = \frac{x - \mu}{\sigma}$$

distribution. Excess kurtosis = Kurtosis - 3. **Leptokurtic**: more peaked. **Platykurtic**: less peaked, flatter.

- **Normal distribution: N~(0, 1). Z score:**
- **Standard** $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ **error:** $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ **Confidence interval:**
- **1.65 for 90% confidence intervals; 1.96 for 95% confidence intervals; 2.58 for 99%.**
- **Hypothesis testing: Two-tailed**: H_0: $\mu = 0$, H_A: $\mu \neq 0$. **One-tailed:** H_0: $\mu \leqslant 0$, H_A: $\mu > 0$.
- **Type I error**: Rejection of the null hypothesis when it is actually true. Significance level.
- **Type II error**: Failure to detect the null hypothesis when it is actually false. Power of test.
- **Simple linear regression**: $Y_i = B_0 + B_1 \times X_i + \epsilon_i$
- R^2 measures the "goodness of fit" of the $R^2 = \frac{ESS}{TSS} = 1 - \frac{SSR}{TSS}$ regression.
- **Regression assumption violations:**
  - ✓ **Heteroskedasticity** occurs when the variance of the residuals is not the same across all observations in the sample.
  - ✓ **Multicollinearity** refers to the condition when two or more of the independent variables, or line a combinations of the independent variables, in a multiple regression are highly correlated with each other.
  - ✓ **Serial correlation** refers to the situation in which the residual terms are correlated with one another. $\text{adjusted } R^2 = 1 - \left(1 - R^2\right) \times \frac{n-1}{n-k-1}$

- **Multiple Linear Regression**: $Y_i = B_0 + B_{1i} \times X_i + B_2 \times X_{2i} + \epsilon_i$;
- EWMA $\sigma_n^2 = (1-\lambda) r_{n-1}^2 + \lambda \sigma_{n-1}^2$ model: $\sigma_n^2 = \omega + \alpha r_{n-1}^2 + \beta \sigma_{n-1}^2$ $\omega = \gamma V_L$

  GARCH Model:

  Continuous compounding: $FV_n = PV_0 \times e^{r \times n}$ Discrete compounding: $FV_n = PV_0 \left(1 + \frac{r}{m}\right)^{m \times n}$

Financial Markets and Products

-
- **Forward** $(1 + \text{forward rate})^t = \frac{(1 + \text{periodic yield})^{t+1}}{(1 + \text{periodic yield})^t}$ **rate:**
- Coupon rate > YTM, bond price > par value, premium bond.

  Coupon rate < YTM, bond price < par value, discount bond

  Coupon rate = YTM, bond price = par value, par bond
- $\text{effective duration} = \frac{BV_{-\Delta y} - BV_{+\Delta y}}{2 \times BV_0 \times \Delta y}$ $\text{percentage bond price change} = -\text{duration} \times \Delta y + \frac{1}{2} \times \text{convexity} \times \Delta y^2$
- **Callable bond**: issuer has the right to buy back the bond. Decreasing rate, negative convexity.

  Putable bond: buyer has the right to sell back the bond.
- **Expected loss** $UL = AE \times \sqrt{EDF \times \sigma_{LGD}^2 + LGD^2 \times \sigma_{EDF}^2}$ = exposure × loss given default × probability of default
- **Unexpected loss** : $HR = \rho_{S,F} \frac{\sigma_S}{\sigma_F}$
- **Minimum variance hedge** $\#\text{ of contracts} = \beta_P \times \left(\frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}} \right)$ **ratio**:
- **Hedging with stock index futures**:
- **Forward Rate Agreement:**

  Cash flow(if paying R_K)=$L*(R-R_K)*(T_2-T_1)$; cash flow(if receiving R_K)=$L*(R_K-R)*(T_2-T_1)$;

- **Cost-of-Carry Model:**

  $F_0 = S_0 e^{rT}$ $F_0 = (S_0 + U - I)e^{rT}$ $F_0 = (S_0 + U - I)e^{rT}$

- **Backwardation: Future price < spot price; Contango: Future price > spot price**

- **Cheapest-to-delivery(CTD):**

  Cash received by the short=quoted futures price × conversion factor + accrued interest

  Cash to purchase bond = quoted bond price + accrued interest

  $$AI = \text{coupon} \times \left(\frac{\text{\# of days from last coupon to the settlement date}}{\text{\# of days in coupon period}} \right)$$

- **Duration-Based hedge ratio:** $\#\ \text{of contracts} = -\frac{\text{portfolio value} \times \text{duration}_P}{\text{futures value} \times \text{duration}_F}$

- **Put-Call Parity**: $P+S=C+Xe^{-rt}$

- Covered call = long stock+ short call; protected put=long stock + long put

Valuation and Risk Models

- **Value at Risk**: Minimum amount one could expect to lose. $VaR(X\%) = z_{X\%} \times \sigma$

- **Binomial Option Pricing Model** (two-period binomial model)

  Step 1: Calculate option payoff at the end of all states.

  Step 2: Calculate option values using risk-neutral probabilities.

  $\text{size of up move} = U = e^{\sigma\sqrt{T}}$ $\text{size of down move} = D = 1/U$ $\pi_{up} = \frac{e^{rt} - D}{U - D}$; $\pi_{down} = 1 - \pi_{up}$

  Step 3: Discount to today using risk-free rate.

- **Black-Scholes-Merton Model**: $c = S_0 \times N(d_1) - Xe^{-rt} N(d_2)$ $p = Xe^{-rt} N(-d_2) - S_0 \times N(-d_1)$

- Greeks：

- ✓ Delta: estimates the change in value for an option for a one-unit change in stock price.
- ✓ Theta: time decay; change in value of an option for a one-unit change in time.
- ✓ Gamma: rate of change in delta as underlying stock price changes.
- ✓ Vega: change in value of an option for a one-unit change in volatility.
- ✓ Rho: sensitivity of option's price to changes in the risk-free rate.